Inhomogeneous chiral condensates within the Functional Renormalisation Group

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[3]CRC TransRegio 211, Strong-Interaction Matter under Extreme Conditions

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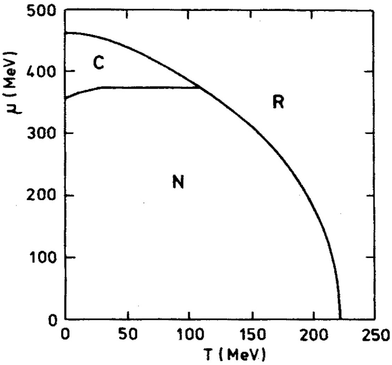
Outline

- Motivation and introduction
- Inhomogeneous chiral condensates within the FRG framework
- FRG based mean-field calculations - Part I - *'the ~~naïve~~ wrong way'*
  - Homogeneous and inhomogeneous chiral condensates
- FRG based mean-field calculations - Part II - *'the ~~consistent~~ better way'*
  - Consistent parameter fixing
  - Aspects of renormalization group consistency
  - Conclusion: The phase diagram(s)
- Summary and outlook

Mean-field phase diagram for the Quark-Meson model (QMM)[1]

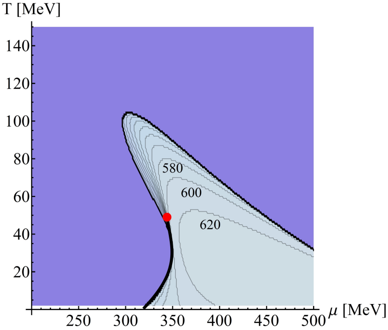


- **N**on-vanishing, homogeneous condensate: $\langle\bar{\psi}\psi\rangle\cancel{(\vec{x})} > 0$
- **R**estored phase with a vanishing homogeneous condensate: $\langle\bar{\psi}\psi\rangle\cancel{(\vec{x})} = 0$
- **C**hiral density wave a non-vanishing, inhomogeneous condensate: $\langle\bar{\psi}\psi\rangle(\vec{x}) > 0$

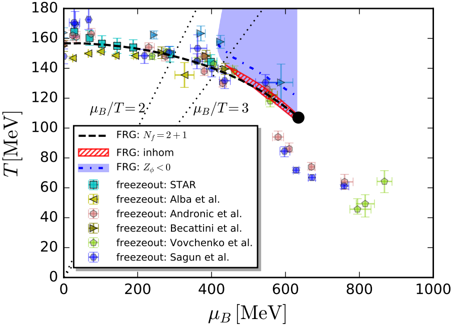
[1] W. Broniowski, A. Kotlorz, and M. Kutschera, Acta Phys. Polon. B **22**, 145–166 (1991).
[2] M. Buballa and S. Carignano, Prog. Part. Nucl. Phys. **81**, 39–96 (2015).

FRG based stability analysis of the homogeneous phase

$N_f = 2$ flavor QMM[3]



$N_f = 2 + 1$ flavor QCD[4]



[3] R.-A. Tripolt, B.-J. Schaefer, et al., Phys. Rev. D. **97**, 034022 (2018).
[4] W.-j. Fu, J. M. Pawlowski, and F. Rennecke, Phys. Rev. D **101**.5, 054032 (2020).

FRG study of inhomogeneous chiral condensates

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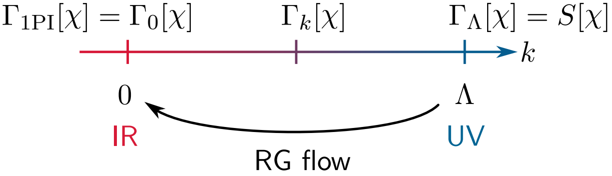
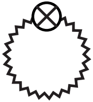
- **Motivation:** Open questions regarding the stability of inhomogeneous chiral condensates under quantum and thermal fluctuations
- **Current goal:** Study effects of bosonic and fermionic quantum fluctuations on inhomogeneous chiral condensates in the QMM
  - $N_f = 2$ quark-meson model in the chiral limit
  - Chiral density wave (CDW) ansatz for the inhomogeneous chiral condensate
- **Method:** Study within the Functional Renormalization Group (FRG)
  - Highly potent tool to investigate effects of quantum fluctuations
  - In-medium computations ($T \geq 0$ and $\mu \geq 0$) are possible
  - Inclusion of inhomogeneous condensates is formally unproblematic

Functional Renormalization Group (FRG)

- Implementation of Wilson's RG approach[5]:



- Exact renormalization group equation[6]:

$$\frac{\mathrm{d}\overline{\Gamma}_k[\chi]}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{STr}\Big\{\big[\overline{\Gamma}_k^{(2)}[\chi] + R_k\big]^{-1}\,\partial_k R_k\Big\} = \frac{1}{2}$$ 

[5] C. Wilson, Phys. Rev. B **4** 9, 3174–3183 (1971).
[6] C. Wetterich, Phys. Lett. B **301** 1, 90–94 (1993).

- Truncation of $\overline{\Gamma}_k$ is necessary to explicitly solve the flow equation: **Local potential approximation (LPA) for QM model** in the chiral limit:

$$\overline{\Gamma}_k[\psi, \bar{\psi}, \phi] = \int \mathrm{d}^4 z \Big\{ \bar{\psi}(z) \Big[\partial\!\!\!/ + \gamma_0 \mu + g\big(\sigma(z) + \mathrm{i}\gamma_5 \vec{\tau} \cdot \vec{\pi}(z)\big) \Big] \psi(z) + \\ + \frac{1}{2}\big(\partial_\mu \phi(z)\big)\big(\partial^\mu \phi(z)\big) + U_k\big(\phi(z)^2/2\big) \Big\}$$

- **Chiral density wave (CDW) ansatz for the condensates:**

$$\phi(z) \overset{\mathrm{CDW}}{=} \big(\sigma(\vec{z}), 0, 0, \pi_3(\vec{z})\big) = \frac{M}{g} \big(\cos(\vec{q} \cdot \vec{z}), 0, 0, \sin(\vec{q} \cdot \vec{z})\big)$$

$$2\rho(\not{z}) \equiv \phi(z)\phi(z) \overset{\mathrm{CDW}}{=} \frac{M^2}{g^2}$$ Spatially independent $O(4)$-sym. field

$$\sigma(z) \pm \mathrm{i} O \pi_3(z) \overset{\mathrm{CDW}}{=} \frac{M}{g} \exp\left(\pm \mathrm{i} O\, \vec{q} \cdot \vec{z}\right), \quad \text{for } O^2 = \mathbb{1}$$ *Euler's formula*

Two-point functions with CDW condensates

- **Challenge:** Non-trivial position dependence for the CDW in

$$\overline{\Gamma}_k^{(0,1,1)}(x,y) \equiv \frac{\delta}{\delta\psi(y)}\frac{\delta}{\delta\bar{\psi}(x)}\overline{\Gamma}_k[\psi,\bar{\psi},\phi]$$
$$\overset{\text{CDW}}{=} \delta^{(4)}(x-y)\Big[\not{\partial}_x + \gamma_0\mu + M\left(\cos(\vec{q}\cdot\vec{x}) + \mathrm{i}\gamma_5\tau_3\sin(\vec{q}\cdot\vec{x})\right)\Big]$$
$$= \delta^{(4)}(x-y)\Big[\not{\partial}_x + \gamma_0\mu + M\exp\left(\mathrm{i}\gamma_5\tau_3\vec{q}\cdot\vec{x}\right)\Big]$$

$$\overline{\Gamma}_k^{(2,0,0)}(x,y) \equiv \frac{\delta}{\delta\phi_i(x)}\frac{\delta}{\delta\phi_j(y)}\overline{\Gamma}_k[\psi,\bar{\psi},\phi]$$
$$\overset{\text{CDW}}{=} \delta^{(4)}(x-y)\Big[\left(-\partial_x^2 + U_k'(\rho)\right)\delta_{ij} + U_k''(\rho)\phi_i(x)\phi_j(x)\Big]$$

- **Solution:** Construct unitary transformation ($U^\dagger U = \mathbb{1}$ and $\partial_k U = 0$) for the CDW analytically to eliminate explicit position dependence $\Leftrightarrow$ diagonalize $\overline{\Gamma}_k^{(2)}$ in momentum space[7]

[7] M. J. Steil, M. Buballa, and B.-J. Schaefer, in preparation.

Unitary transformations for the CDW

- **The transformation for the fermionic two-point function:**

$$U_F(\vec{x}) \equiv \exp\big(-\frac{\mathrm{i}}{2}\gamma_5\tau_3\vec{q}\cdot\vec{x}\big)$$

diagonalizes $\gamma_0\overline{\Gamma}_k^{(0,1,1)}$ in momentum space[8].

- **The transformation for the bosonic two-point function:**

$$U_B(\vec{x}) \equiv \frac{1}{2}\begin{pmatrix} 1-\exp(-2\mathrm{i}\vec{q}\cdot\vec{x}) & 0 & 0 & 1+\exp(-2\mathrm{i}\vec{q}\cdot\vec{x}) \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ -\mathrm{i}(1+\exp(-2\mathrm{i}\vec{q}\cdot\vec{x})) & 0 & 0 & \mathrm{i}(\exp(-2\mathrm{i}\vec{q}\cdot\vec{x})-1) \end{pmatrix}.$$

diagonalizes $\overline{\Gamma}_k^{(2,0,0)}$ in momentum space.[7]

[8]F. Dautry and E. M. Nyman, Nucl. Phys. A **329** 3, 491–523 (1979).
[7]M. J. Steil, M. Buballa, and B.-J. Schaefer, in preparation.

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LPA flow equation for $U_k(\rho)$ with CDW condensates

$$\partial_k U_k(\rho) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{i=0}^{3} \frac{1}{2} \coth\left(\frac{E_k^i}{2T}\right) \tilde{\partial}_k E_k^i + \\ - 2N_c \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \sum_{\pm,\pm} \tanh\left(\frac{E_k^{\pm} \pm \mu}{2T}\right) \tilde{\partial}_k E_k^{\pm}$$

- Using generic but **three-dimensional** FRG regulators

$$R_k^F(p,p') \equiv -\mathrm{i}\vec{\not{p}} r_k^F(|\vec{p}|/k)(2\pi)^4 \delta^{(4)}(p-p')$$
$$R_k^B(p,p') \equiv \vec{p}^{\,2} r_k^B(|\vec{p}|/k)(2\pi)^4 \delta^{(4)}(p-p')$$

in a unified regulator scheme

$$\left(1 + r_k^F(|\vec{p}|/k)\right)^2 = 1 + r_k^B(|\vec{p}|/k) \equiv \left(\lambda_k(|\vec{p}|)\right)^2.$$

Flowing energy eigenvalues of the CDW

- **Fermionic eigenvalues**

$$(E_k^{\pm})^2 = M^2 + \frac{(\vec{p}_k^{\,+q})^2}{2} + \frac{(\vec{p}_k^{\,-q})^2}{2} + \pm\sqrt{M^2\left(\vec{p}_k^{\,+q} - \vec{p}_k^{\,-q}\right)^2 + \frac{1}{4}\left((\vec{p}_k^{\,+q})^2 - (\vec{p}_k^{\,-q})^2\right)^2}$$

$$\overset{q=0}{=} M^2 + (\vec{p}_k)^2$$

with $\vec{p}_k^{\,q} \equiv \left(\vec{p} + \vec{q}/2\right)\left(1 + r_k^F(|\vec{p} + \vec{q}/2|/k)\right) = \left(\vec{p} + \vec{q}/2\right)\lambda_k(|\vec{p} + \vec{q}/2|)$

- **Bosonic eigenvalues**

$$(E_k^1)^2 = (E_k^2)^2 = (\vec{p}_k)^2 + U_k'(\rho) \overset{q=0}{=} (\vec{p}_k)^2 + U_k'(\rho)$$

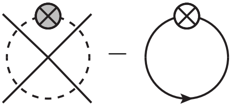
$$(E_k^{0,3})^2 = \frac{1}{2}(\vec{p}_k)^2 + \frac{1}{2}(\vec{p}_k^{\,+4q})^2 + U_k'(\rho) + \rho\, U_k''(\rho) + \pm\sqrt{\rho^2 U_k''(\rho)^2 + \frac{1}{4}\left((\vec{p}_k^{\,+4q})^2 - (\vec{p}_k)^2\right)^2}$$

$$\overset{q=0}{=} (\vec{p}_k)^2 + U_k'(\rho) + \rho(U_k''(\rho) \pm |U_k''(\rho)|)$$

FRG based mean-field calculations - Part I

- **Mean-field approximation (MFA)** in the present RG setting: Neglect bosonic fluctuations and integrate the LPA flow equation.

$$\partial_k \overline{\Gamma}_k = \frac{1}{2}$$



  - UV initial condition

$$U_\Lambda(\rho) = \lambda_\Lambda \rho^2 + m_\Lambda^2 \rho = \lambda_\Lambda(\rho + v_\Lambda^2)\rho$$

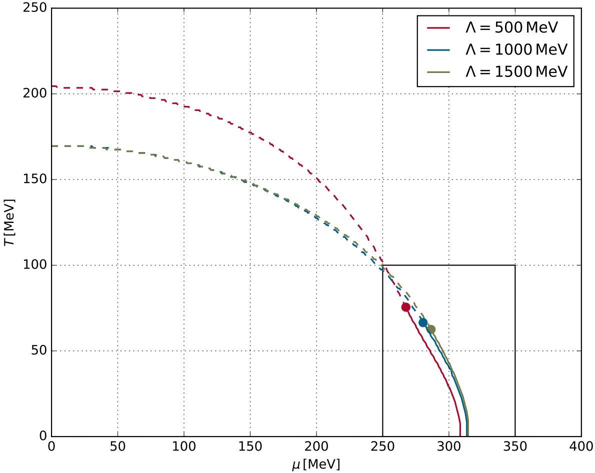
  - 3D/spatial exponential regulator shape function

$$\left(1 + r_k^F(|\vec{p}|/k)\right)^2 = \left(\exp(\vec{p}^{\,2}/k^2) - 1\right)^{-1} + 1$$

  - Model parameters $(g, \lambda_\Lambda, m_\Lambda)$ are fitted by fixing the bare pion decay constant f_π^{b}, the curvature mass of the sigma meson m_σ^{c} and the quark mass M_ψ to 'physical' values

Homogeneous RG MF phase diagrams

$f_\pi^{\mathrm{b}} = 88\,\mathrm{MeV},\ M_\psi = 300\,\mathrm{MeV}$ and $m_\sigma^{\mathrm{c}} = 600\,\mathrm{MeV}$



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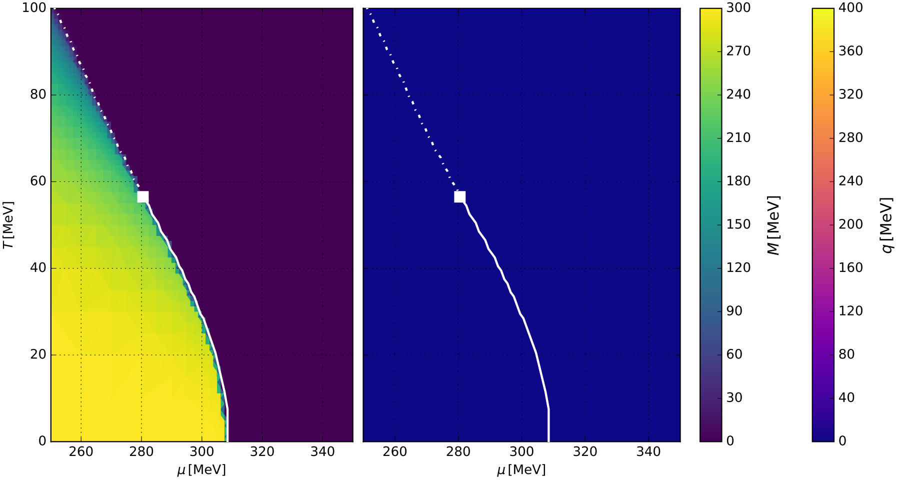
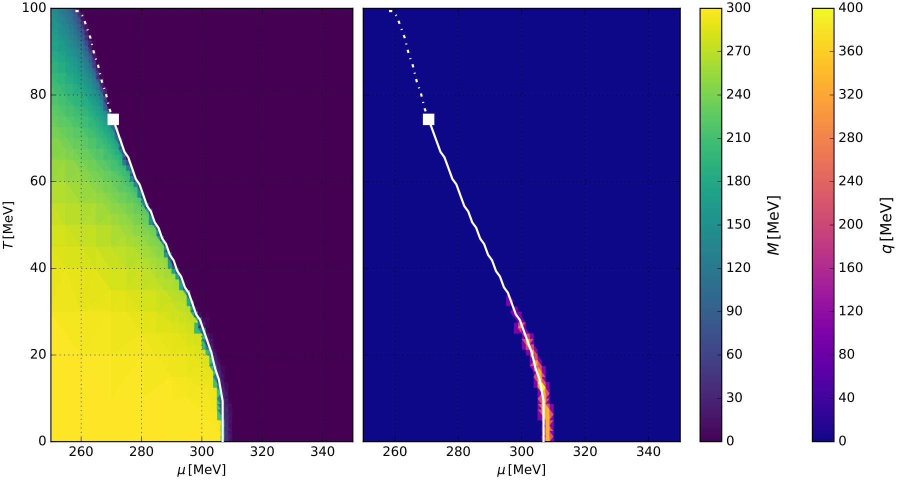
$f_\pi^{\mathrm{b}} = 88\,\mathrm{MeV}$, $M_\psi = 300\,\mathrm{MeV}$ and $m_\sigma^{\mathrm{c}} = 600\,\mathrm{MeV}$, $\Lambda = 500\,\mathrm{MeV}$



Inhomogeneous RG MF phase diagrams

$f_\pi^{\mathrm{b}} = 88\,\mathrm{MeV}$, $M_\psi = 300\,\mathrm{MeV}$ and $m_\sigma^{\mathrm{c}} = 600\,\mathrm{MeV}$, $\Lambda = 450\,\mathrm{MeV}$



Inhomogeneous RG MF phase diagrams

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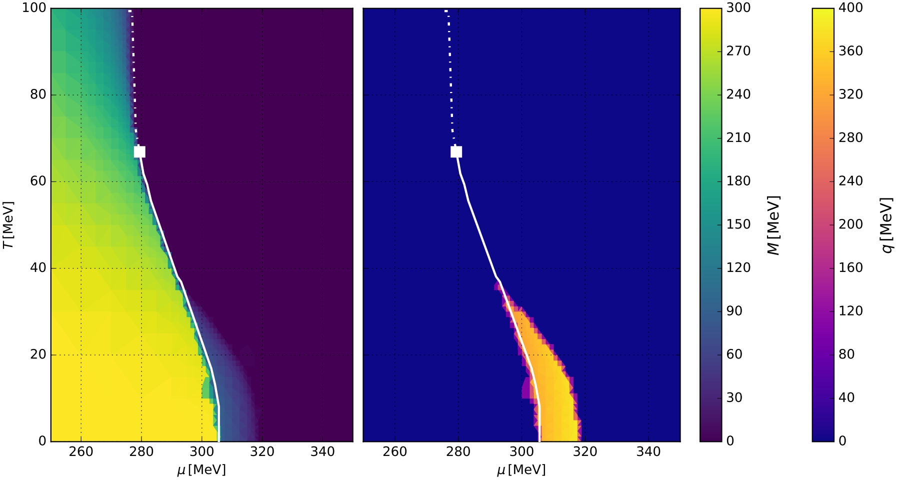
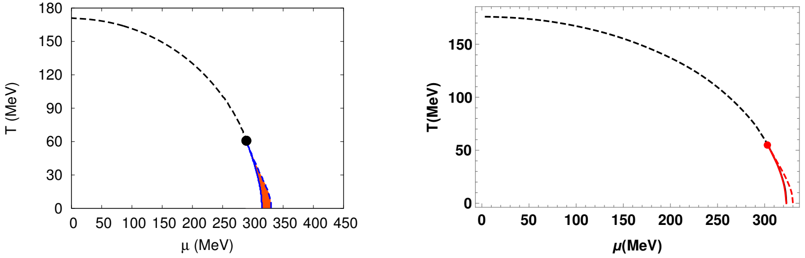
$$f_\pi^{\mathrm{b}} = 88\,\mathrm{MeV},\ M_\psi = 300\,\mathrm{MeV} \text{ and } m_\sigma^{\mathrm{c}} = 600\,\mathrm{MeV},\ \Lambda = 400\,\mathrm{MeV}$$



- Involved existing MF results (with $M_\psi = 300\,\mathrm{MeV}$, $m_\sigma = 2M_\psi$)
  - PV regularization and 'RP' parameter fixing at $\Lambda_{\mathrm{PV}} = 5.0\,\mathrm{GeV}$[9]
  - Dim. regularization using the on-shell (OS) renormalization scheme[10]

are in agreement and predicts a **non-vanishing inhomogeneous window**:

PV 'RP': $f_\pi = 88\,\mathrm{MeV}$ **dim. reg. 'OS':** $f_\pi = 93\,\mathrm{MeV}$



[9] S. Carignano, M. Buballa, and W. Elkamhawy, Phys. Rev. D **94** 3, 034023 (2016).
[10] P. Adhikari, J. O. Andersen, and P. Kneschke, Phys. Rev. D **96** 1, 016013 (2017).

Improving on the naïve RG MFA: RG MF - Part II

- Improved/consistent parameter fixing using $\Gamma^{(2)}_{k=0}$ in MFA
  - Fitting renormalized pion decay constant f_π^{r} (not f_π^{b})
  - Fitting pole-mass m_σ^{p} (not m_σ^{c})
  - Motivated by MF studies with Pauli-Villars regularization[9]

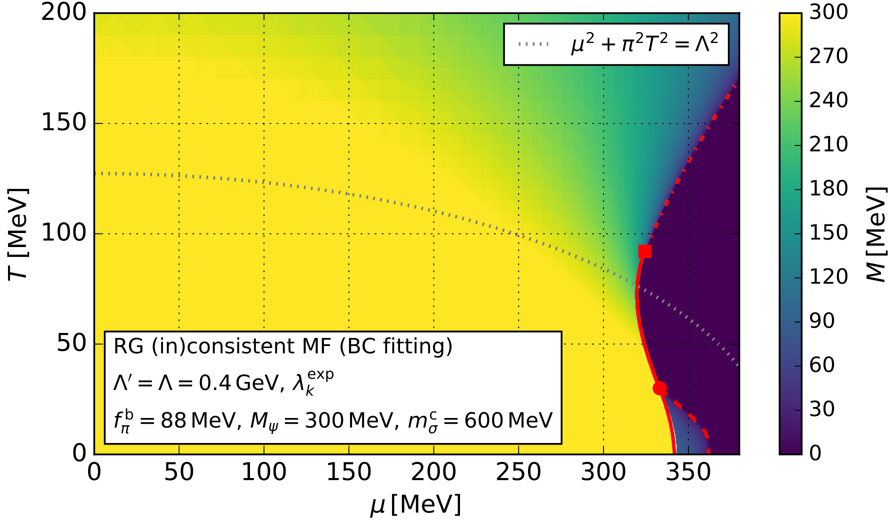
- *RG-consistent* MFA[11] by enforcing:

$$\Lambda \frac{\mathrm{d}\Gamma_{k=0}}{\mathrm{d}\Lambda} = 0 \qquad \forall\, T, \mu$$

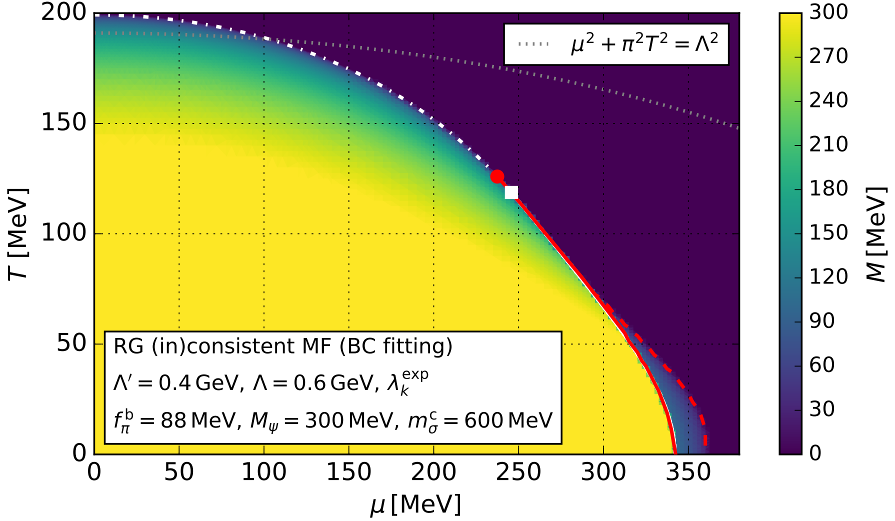
  - Initial condition $\Gamma_{\Lambda'}[\rho]$ at $\Lambda' < \Lambda$ and construction of $\Gamma_\Lambda[\rho]$ via RG-consistency (flow eq.) $\Rightarrow$ Systematic UV completion $\forall\, T, \mu$
  - Allows for systematic study of cutoff effects and regularization-scheme dependence
  - Practical implementation on MF-level is simple

[9] S. Carignano, M. Buballa, and W. Elkamhawy, Phys. Rev. D **94** 3, 034023 (2016).
[11] J. Braun, M. Leonhardt, and J. M. Pawlowski, SciPost Phys. **6**, 056 (2019).

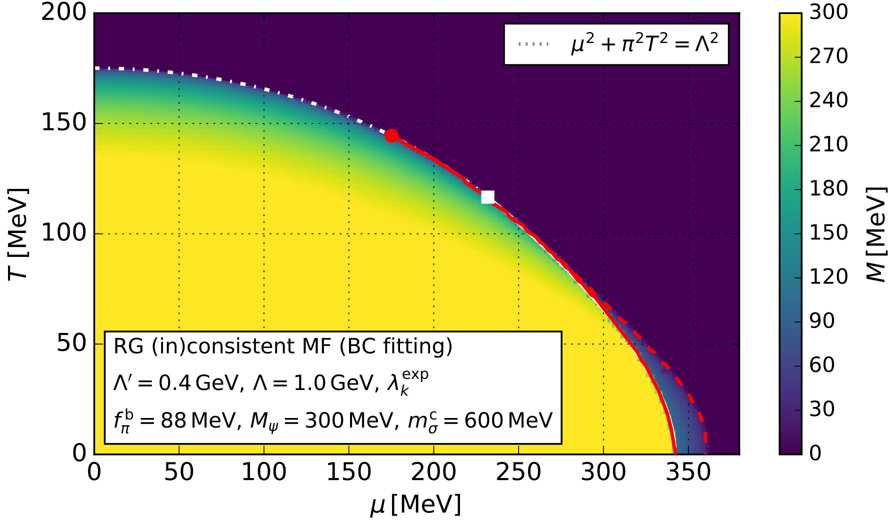
RG (in)consistency and the phase diagram



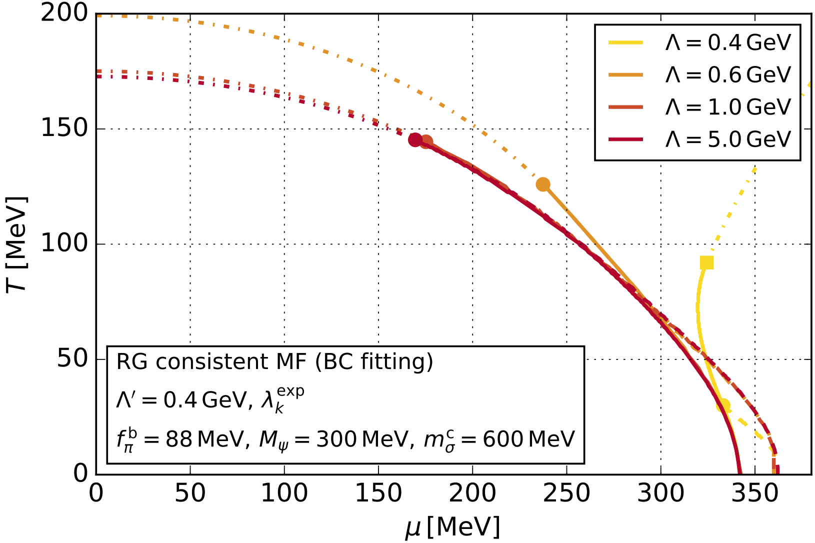
RG (in)consistency and the phase diagram



RG (in)consistency and the phase diagram



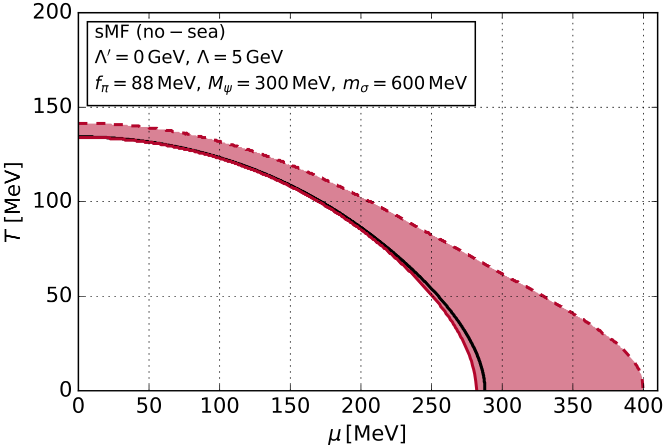
RG (in)consistency and the phase diagram



No-sea limit in RG consistent MF computations

$\Lambda' \to 0 \Rightarrow \Gamma_{\Lambda'}[\rho]$ includes not loop contributions

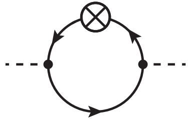
$\Lambda \to \infty \Rightarrow$ FRG regulator $r_k^F(|\vec{p}|/k)$-dependency drops out



- Evaluating the flow eq. of the bosonic two-point function on the MF RG flow in vacuum at the physical minimum yields:

$$\frac{\mathrm{d}}{\mathrm{d}k}\Gamma_k^{\phi\phi}(p_\mathrm{I}^0,\vec{p}_\mathrm{I}) = 2\,G_{k;\psi\bar{\psi}}\Gamma^{,\bar{\psi}\phi\psi}G_{k;\psi\bar{\psi}}\Gamma^{,\bar{\psi}\phi\psi}G_{k;\psi\bar{\psi}}\partial_k R_k^{\bar{\psi}\psi}$$

$$= 2$$



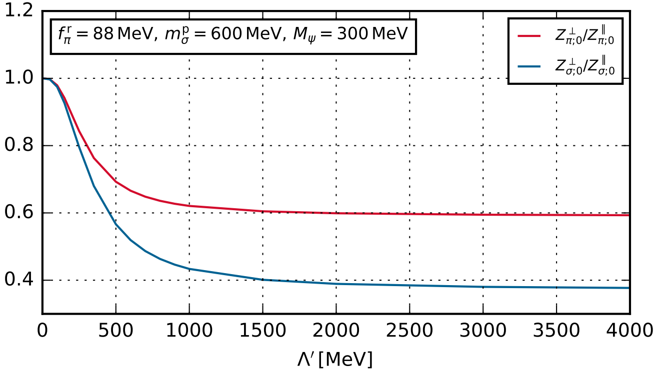
- Retarded 2-point function:

$$\Gamma_\phi^{(2),R}(\omega,\vec{p}) = \lim_{\epsilon\to 0}\Gamma_0^{\phi\phi}(p_\mathrm{I}^0 = -\mathrm{i}(\omega+\mathrm{i}\epsilon),\vec{p})$$
$$= -\omega^2 + \vec{p}^2 + 2\lambda_{\Lambda'}(1+2\delta_{\phi\sigma})\rho + m_{\Lambda'}^2 + L_\phi^{\Lambda'}(\omega,\vec{p})$$

Wave function renormalizations at $T = \mu = 0$

$$Z^{\parallel}_{\phi;0} = -\frac{1}{2}\left(\frac{\partial^2}{\partial\omega^2}\mathrm{Re}\,\Gamma^{(2),R}_{\phi}(\omega,\vec{p})\right)_{\omega=0,\,\vec{p}=0}$$

$$Z^{\perp}_{\phi;0} = \frac{1}{2}\left(\frac{\partial^2}{\partial\vec{p}^2}\mathrm{Re}\,\Gamma^{(2),R}_{\phi}(\omega,\vec{p})\right)_{\omega=0,\,\vec{p}=0}$$



Poincaré-invariance 'beyond' LPA

Why do we find the splitting $Z^{\parallel}_{\phi;0} \neq Z^{\perp}_{\phi;0}$ in vacuum in the IR?

$$\Uparrow$$

Because a regularization-scheme using three-dimensional/spatial regulators breaks Poincaré-invariance explicitly!

Solutions:

- (Switch to covariant/four-dimensional regulators)
- Enforce $Z^{\parallel}_{\phi;0} \stackrel{!}{=} Z^{\perp}_{\phi;0}$ in the IR by an appropriate choice of $Z^{\parallel}_{\phi;\Lambda'}$ with $Z^{\perp}_{\phi;\Lambda'} = 1$ in the UV[12] **(RP)**
- Live with it: Use $Z^{\perp}_{\pi;0}$ and accept deviations for m^{p}_{σ} **(RP*)**

[12] J. Braun, Phys. Rev. **D81**, 016008 (2010).

Consistent (RP/RP*) parameter fixing

- Consistent scheme: including fermionic vacuum fluctuations by fitting the **r**enormalized pion-decay constant f_π^{r}, the sigma **p**ole mass m_σ^{p} and the quark mass M_ψ to 'physical' values
  - We define the sigma pole mass m_σ^{p} as

    $$0 = \mathrm{Re}\,\Gamma_\sigma^{(2),R}(m_\sigma^{\mathrm{p}}, \vec{0}) = -Z^{\parallel}_{\sigma;\Lambda'}(m_\sigma^{\mathrm{p}})^2 + 6\lambda_{\Lambda'}\rho + m^2_{\Lambda'} + \mathrm{Re}\,L_\sigma^{\Lambda'}(m_\sigma^{\mathrm{p}}, \vec{0}),$$

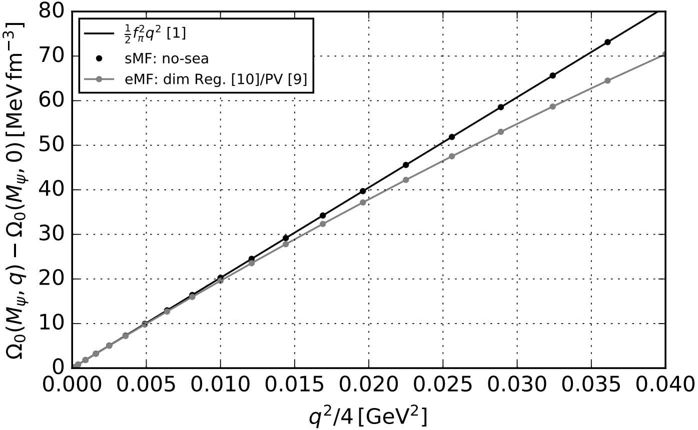
    where $Z^{\parallel}_{\sigma;\Lambda'}$ is chosen to realize $Z^{\parallel}_{\sigma;0} = Z^{\perp}_{\sigma;0}$ in the IR **(RP)** or $Z^{\parallel}_{\sigma;\Lambda'} = 1$ **(RP*)**
  - For the renormalized pion-decay constant

    $$f_\pi^{\mathrm{r}} = \left(Z^{\perp}_{\pi;0}\right)^{1/2} f_\pi^{\mathrm{b}}$$

    we extract the wave function renormalization from

    $$Z^{\perp}_{\pi;0} = \frac{1}{2}\left(\frac{\partial^2}{\partial \vec{p}^2}\mathrm{Re}\,\Gamma_\pi^{(2),R}(\omega, \vec{p})\right)_{\omega=0,\,\vec{p}=0}.$$

Existing MF results



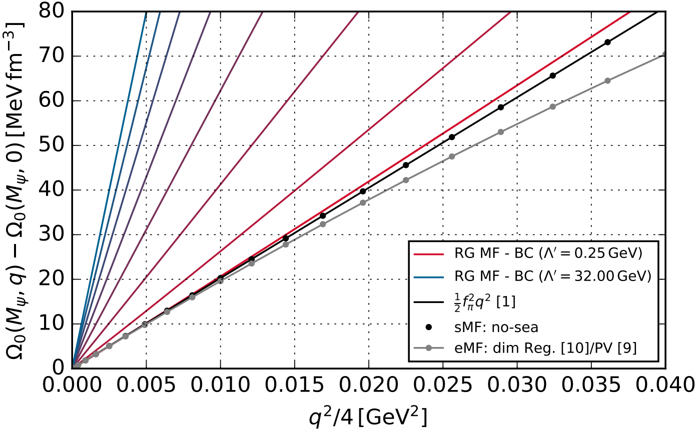
[1] W. Broniowski, A. Kotlorz, and M. Kutschera, Acta Phys. Polon. B **22**, 145–166 (1991).

[9] S. Carignano, M. Buballa, and W. Elkamhawy, Phys. Rev. D **94** 3, 034023 (2016).

[10] P. Adhikari, J. O. Andersen, and P. Kneschke, Phys. Rev. D **96** 1, 016013 (2017).

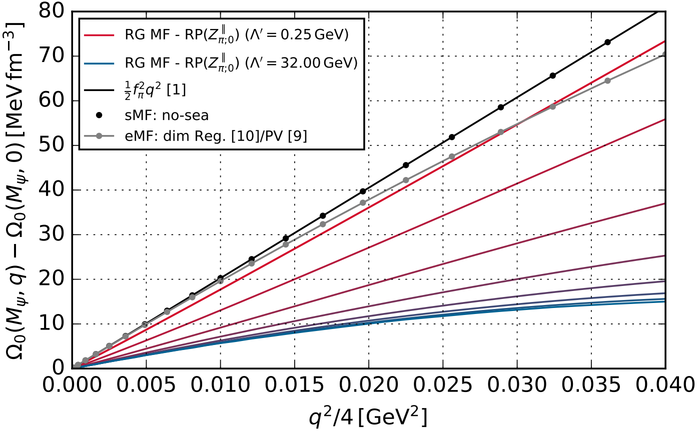
CDW vs. homogeneous ground state at $T = \mu = 0$

RG MF using naïve BC parameter fitting



[1] W. Broniowski, A. Kotlorz, and M. Kutschera, Acta Phys. Polon. B **22**, 145–166 (1991).
[9] S. Carignano, M. Buballa, and W. Elkamhawy, Phys. Rev. D **94** 3, 034023 (2016).
[10] P. Adhikari, J. O. Andersen, and P. Kneschke, Phys. Rev. D **96** 1, 016013 (2017).

RG MF using RP parameter fitting using $Z^{\parallel}_{\pi;0}$ without $Z^{\parallel}_{\pi;0} \overset{!}{=} Z^{\perp}_{\pi;0}$



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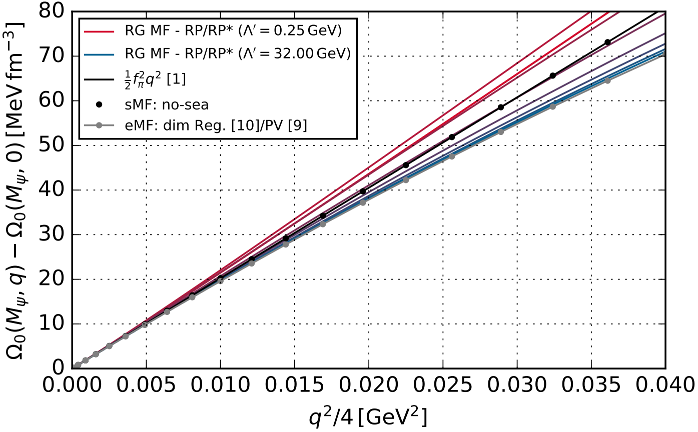
[10] P. Adhikari, J. O. Andersen, and P. Kneschke, Phys. Rev. D **96** 1, 016013 (2017).

CDW vs. homogeneous ground state at $T = \mu = 0$

RG MF using RP/RP* parameter fitting



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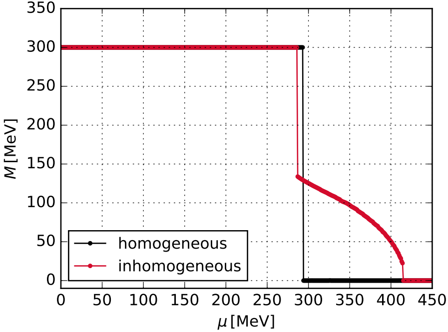
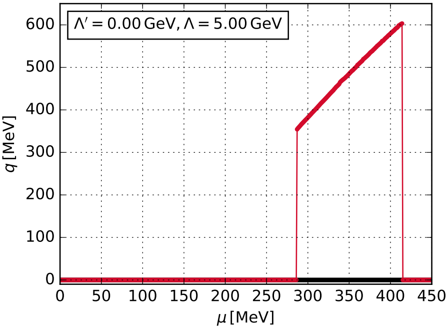
[10] P. Adhikari, J. O. Andersen, and P. Kneschke, Phys. Rev. D **96** 1, 016013 (2017).

Vacuum fluctuations and inhomo. window at $T = 0$

RG consistent MF: $\Lambda' = 0.00\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$ (no-sea)

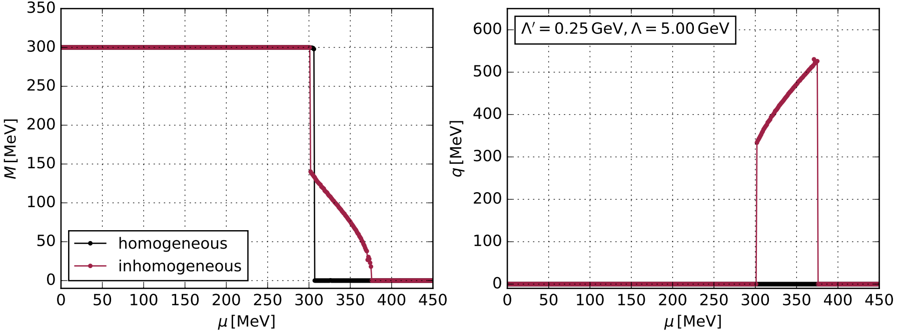
RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$, $m_\sigma^{\mathrm{p}} = 625\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$





RG consistent MF: $\Lambda' = 0.25\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

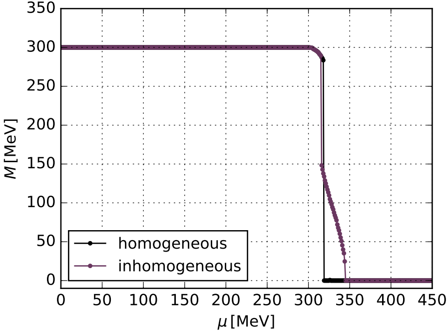
RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$, $m_\sigma^{\mathrm{p}} = 625\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$

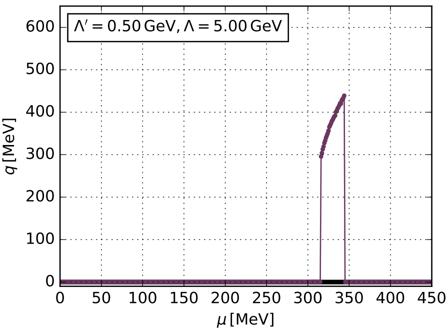


Vacuum fluctuations and inhomo. window at $T = 0$

RG consistent MF: $\Lambda' = 0.50\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

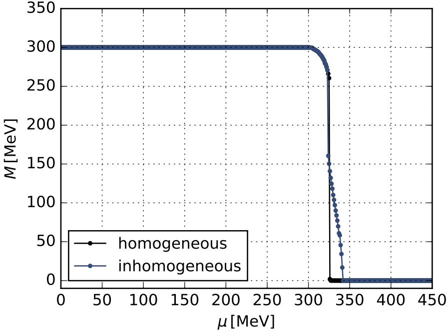
RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$, $m_\sigma^{\mathrm{p}} = 625\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$

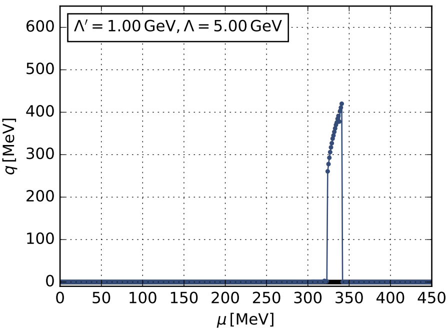




RG consistent MF: $\Lambda' = 1.00\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$, $m_\sigma^{\mathrm{p}} = 625\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$

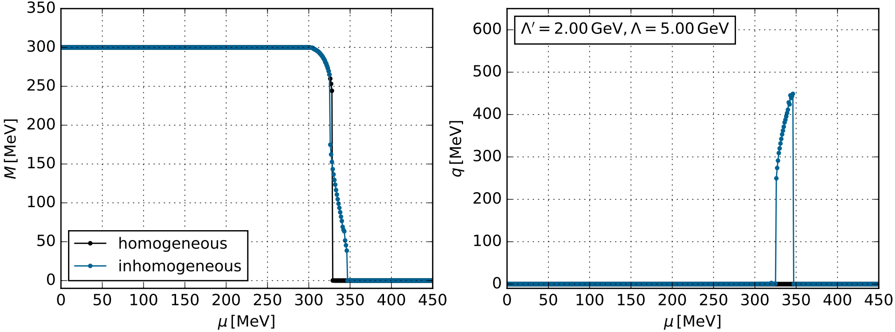




Vacuum fluctuations and inhomo. window at $T = 0$

RG consistent MF: $\Lambda' = 2.00\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

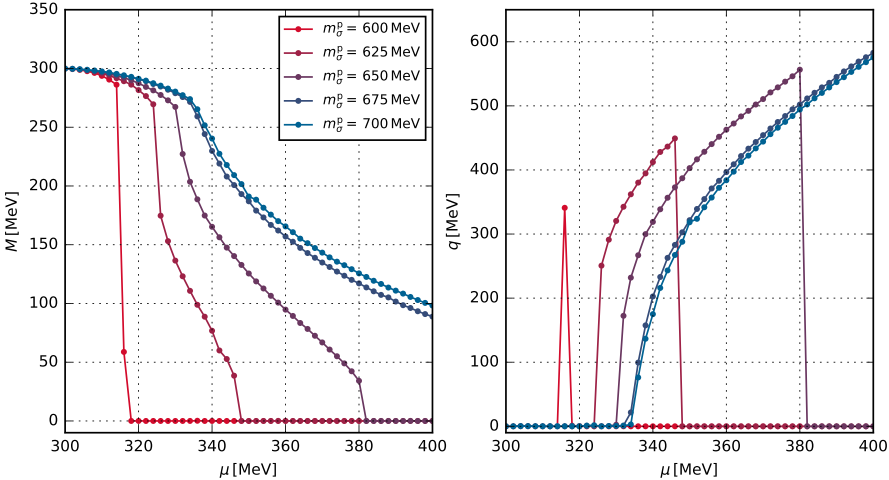
RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$, $m_\sigma^{\mathrm{p}} = 625\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$



CRC-TR 211

RG consistent MF: $\Lambda' = 2.00\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

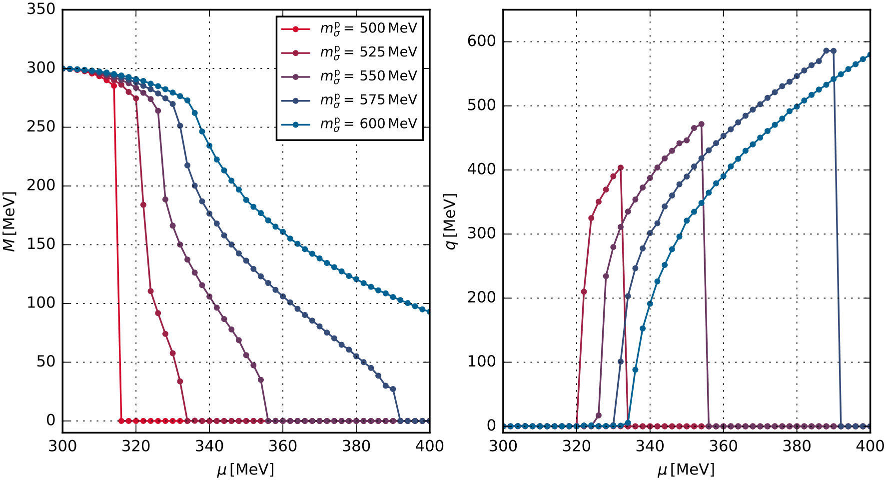
RP parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$



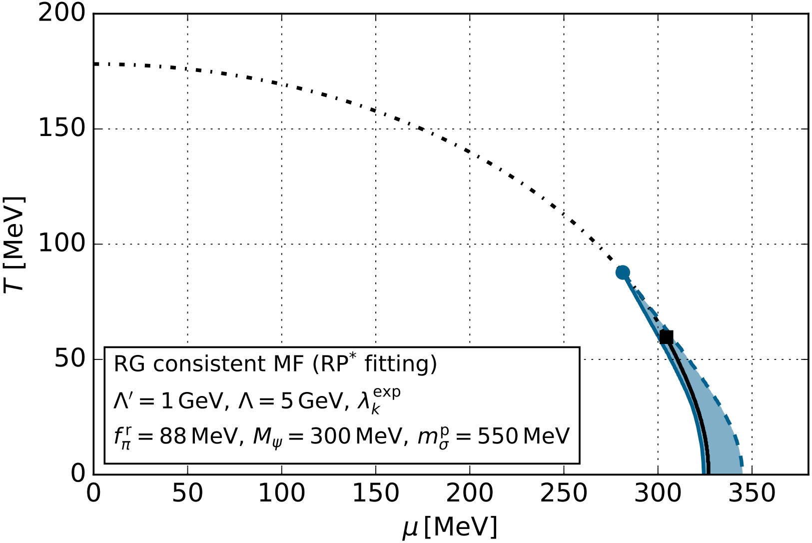
CRC-TR 211

RG consistent MF: $\Lambda' = 2.00\,\mathrm{GeV}$, $\Lambda = 5.00\,\mathrm{GeV}$

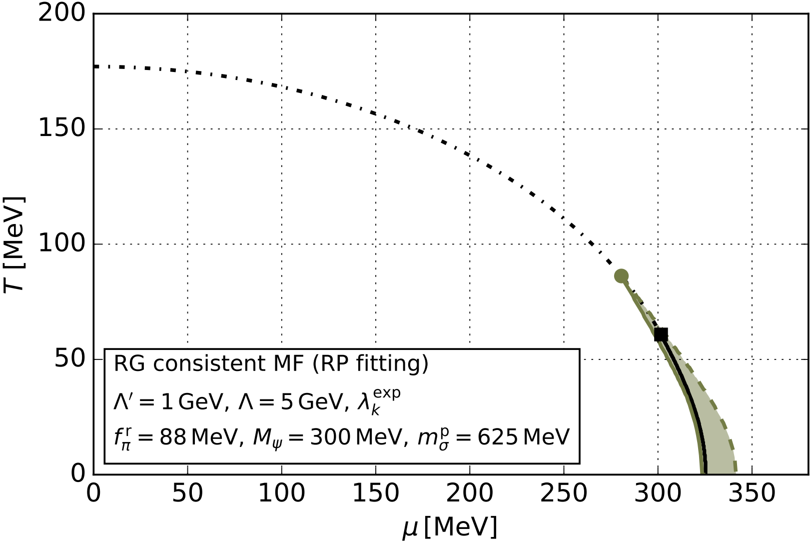
RP* parameter fitting: $f_\pi^{\mathrm{r}} = 88\,\mathrm{MeV}$ and $M_\psi = 300\,\mathrm{MeV}$



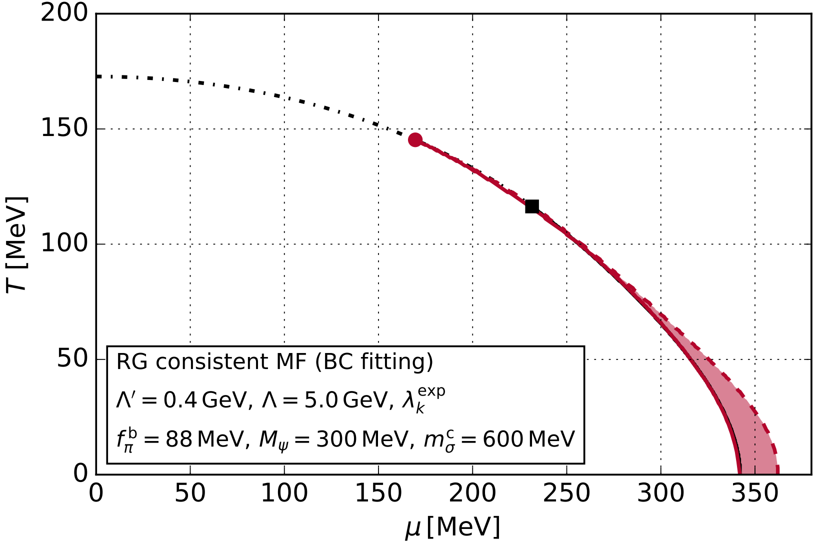
Conclusion: The phase diagram(s)



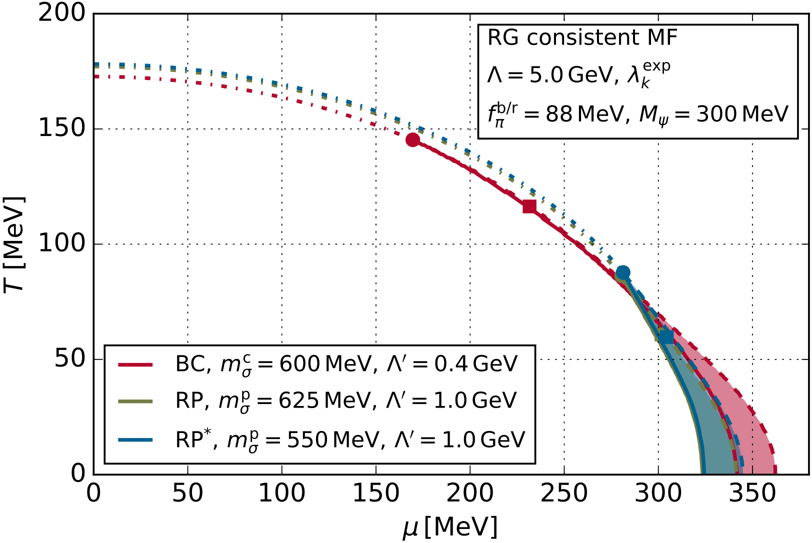
Conclusion: The phase diagram(s)



Conclusion: The phase diagram(s)



Conclusion: The phase diagram(s)



Summary and outlook

- **What we have done so far:**
  - Derivation of a LPA flow eq. for inhomogeneous CDW condensates
  - Numerical results of FRG based mean-field computations
    - RG consistency, fermionic contributions to $\Gamma_k^{\phi\phi}(p_{\mathrm{I}}^0, \vec{p}_{\mathrm{I}}) \Rightarrow f_\pi^{\mathrm{r}}, m_\sigma^{\mathrm{p}}$
    - Qualitative agreement with existing MF results
    - Small quantitative deviation from existing MF results related to m_σ^{p} because of the current regulator choice and the resulting breaking Poincaré-invariance
- **What we are currently working on:**
  - Numerical solution of the full CDW flow equation for the CDW
    - Finite volume methods for discretization in ρ-direction[13] on a q-grid
  - Publication of the presented RG consistent MF results for the CDW
- **What we plan to do in the future:**
  - RG consistent MF study using covariant/four-dimensional regulators
  - Systematic comparison to FRG based stability analysis of the homogeneous phase
  - Extending the truncation: deriving flow equations beyond LPA in presence of CDW condensates

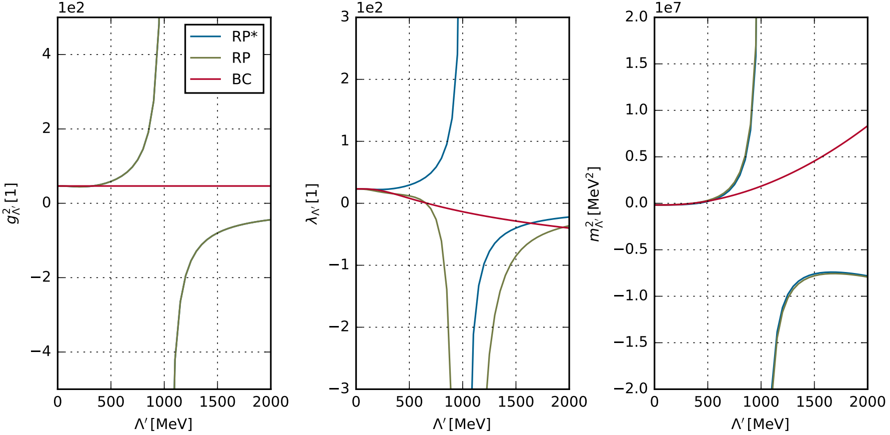
[13] A. Koenigstein, M. J. Steil, et al., in preparation.

xkcd.com/1739/, Online 2020.11.09, 21:00

Appendix

RP* vs RP vs BC parameters over Λ'

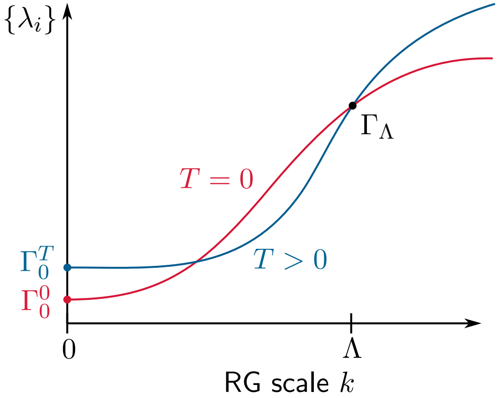
$f_\pi = 88\,\mathrm{MeV}$, $M_\psi = 300\,\mathrm{MeV}$ and $m_\sigma = 600\,\mathrm{MeV}$



CRC-TR 211

- RG consistency is violated at finite T if

$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\Lambda\frac{\mathrm{d}\Gamma_\Lambda}{\mathrm{d}\Lambda}\right) \neq 0 \quad \text{and} \quad \Gamma_\Lambda(\not{T})$$

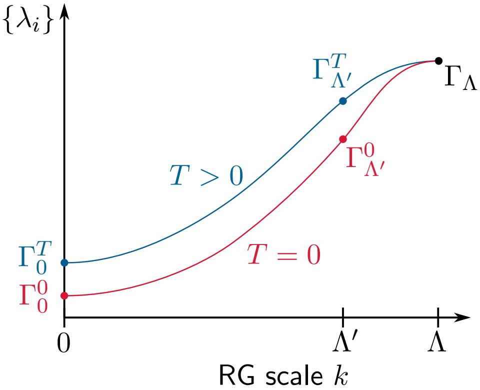


[11] J. Braun, M. Leonhardt, and J. M. Pawlowski, SciPost Phys. **6**, 056 (2019).

RG consistency at finite T (and μ)

- RG consistent construction of Γ_Λ to ensure

$$\frac{\mathrm{d}}{\mathrm{d}T}\left(\Lambda\frac{\mathrm{d}\Gamma_\Lambda}{\mathrm{d}\Lambda}\right) = 0 \quad \Rightarrow \quad \Gamma_{\Lambda'}(T) \text{ for } \Lambda' < \Lambda$$



[11] J. Braun, M. Leonhardt, and J. M. Pawlowski, SciPost Phys. **6**, 056 (2019).

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A. Koenigstein, M. J. Steil, et al., in preparation.